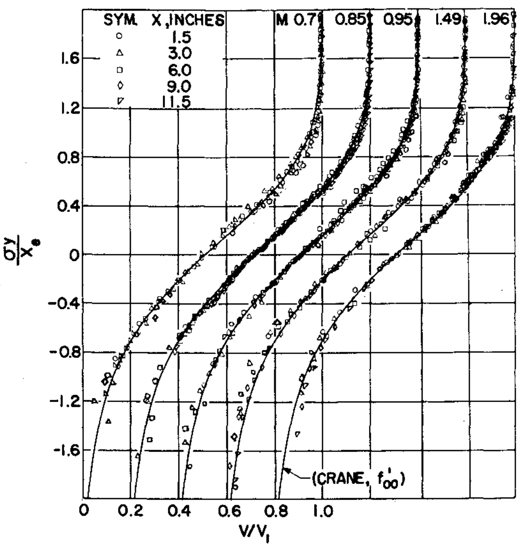

Fig. 1 Normalized velocity profiles

indicate the same results as the data from the 25 surveys shown in Fig. 1.

The nondimensional velocity profiles are similar at axial stations from $0.5 \leq x/D \leq 3.84$ at the five Mach numbers, i.e., there is no discernible effect of Mach number (at $M < 2$) on the shape of the velocity profile. This tends to support Crane's[4] reasoning that (because of compensating effects) the nondimensional velocity profile for compressible flow essentially is unchanged from that obtained for incompressible flow.

These jet-spreading parameter data are presented in Fig. 2, along with the available experimental data[3] from other sources; the estimated variation of σ with M also is shown. A value of $\sigma = 12$ for incompressible flow has been used widely for many years; however, Liepmann and Laufer determined a value of $\sigma = 11$ from comparison of their low-speed data with Gortler's (or Crane's) theoretical velocity profile. Hence, it is concluded that the spreading rate of the mixing region is essentially constant ($\sigma \approx 11$) at subsonic speeds and decreases (with increasing Mach) at supersonic speeds. The estimated error in determining σ in these experiments is $\pm 5\%$.

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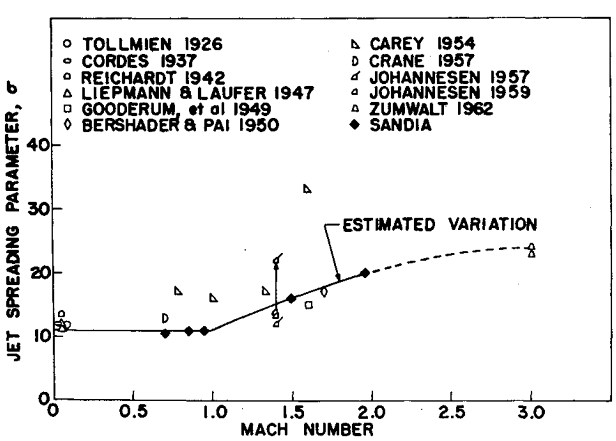

Fig. 2 Summary of jet spreading data

Successful Re-Entry of Space Fragments from a Decaying Earth Orbit

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Nomenclature

A = reference area for drag, ft^2
c = heat capacity, Btu/lb°R
C_D = drag coefficient
g_c = gravitational constant, 32.2 ft/sec^2
R = effective radius, ft
r = actual radius, ft
S = surface wetted by boundary layer, ft^2
T_w = maximum temperature of fragment during re-entry, °R
t = thickness, in.
W = weight, lb
ρ = fragment density, lb/ft^3
ϵ = surface emissivity
σ = Stefan-Boltzmann constant, 0.48×10^{-12} Btu/ft^2 sec °R^4

THE fact that space debris can successfully re-enter the earth's atmosphere has been underlined by the recent orbital decay of a Soviet satellite and of United States space boosters.[1, 2] One of the most publicized Soviet fragments is a thick metal disk weighing about 20 lb. Most of the fragments recovered from the space boosters can be typified as thin metal plates. The idealized fragment whose re-entry heating characteristics are examined in this note is a circular plate oriented perpendicular to the flow.

For cases of negligible heat capacity (very thin plates), Chapman's expression for laminar-convective heat flux[3] may be equated with the radiant heat flux from the fragment to give

$$\epsilon\sigma T_w{}^4 = 590 K_1 \left(\frac{W}{C_D A R g_c}\right)^{1/2} \bar{q} \qquad (1)$$

For re-entry of nonlifting bodies from a decaying earth orbit, the maximum value[3] of the dimensionless heating rate $\bar{q}$ is 0.218. For this disk geometry, the local-convective heat flux will be assumed equal to the stagnation value (cf., Ref. 4) so that $K_1 = 1$.

For heat-sink fragments (very thick plates), one may equate the total convective heat flux during re-entry to the heat stored in the fragment. Equating the expression for total laminar heat flux, found in Ref. 3, to the stored heat gives (neglecting radiation from the surface and internal temperature gradients)

$$Wc(T_w - T_i) = 15{,}900\, K_2 S \left(\frac{W}{C_D A R g_c}\right)^{1/2} \bar{Q} \qquad (2)$$

For nonlifting re-entry from an earth orbit,[3] $\bar{Q} = 1.36$; the assumption $K_1 = 1$ implies $K_2 = 1$.

In evaluating these equations the authors will take $r = 1$, $C_D = 2$, $\rho = 501$, and $c = 0.17$, which is typical for stainless

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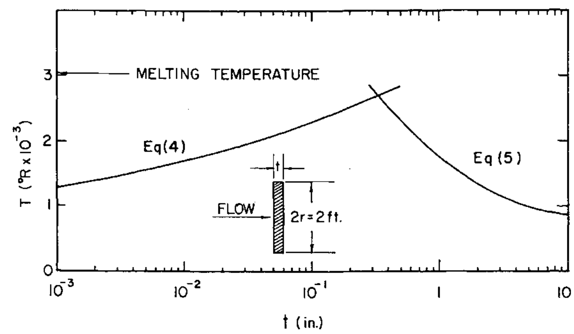

Fig. 1 Calculated maximum temperature T_w of a steel disk of thickness t during re-entry from a decaying earth orbit

steel over the expected temperature range. A value for total emissivity of $\epsilon = 0.6$ was chosen from Ref. 5 for high-temperature steel.

For the particular case of a steel disk, the parameter W/C_DA reduces to

$$(W/C_DA) = (\rho At/C_DA) = 21t \tag{3}$$

in the units given in the Nomenclature. In evaluating Eqs. (1) and (2) it has been assumed that heat is radiated from both sides of the disk, that $R = 4r$ (cf., Ref. 6), and that the initial temperature of the fragment is 500°R. The equations for the maximum temperature of the fragment during re-entry then reduce to

$$T_w = 3070t^{1/8} \tag{4}$$

and

$$T_w = 500 + 1230t^{-1/2} \tag{5}$$

The resulting curves for T_w as a function of t are given in Fig. 1, which shows that the temperature for both the very thin and very thick fragments (corresponding to low and high values of W/C_DA) are lower than in the intermediate-thickness region. In this intermediate-thickness region, the approximations upon which Eqs. (4) and (5) are based do not hold, but the actual value of T_w must lie below the values given by these approximations.

It is most interesting to note that even in the intermediate-thickness region the maximum calculated temperature is still below the melting point of steel, leading to the conclusion that *all* such disks would successfully re-enter the earth's atmosphere. For other materials or for other shapes (such as spheres, hollow spheres, etc.), the curves of maximum temperature vs a characteristic dimension of the body will be qualitatively similar to curves of T_w vs t given in Fig. 1 for steel disks. In other cases, however, an intermediate-size region may exist where the calculated temperatures exceed the melting point of the material, and the object may not re-enter successfully, as found for re-entry at meteor velocities.[7]

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Regions of Libration for a Symmetrical Satellite

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A SMALL, symmetrical rigid body moves in a circular orbit about a large point mass. The gravity gradient across the body, due to the attraction of the large point mass, produces a torque on the body about its mass center. Because of symmetry the component of angular velocity along the symmetry axis (the ζ axis) is a constant that will be taken to be zero. The positions of stable, relative equilibrium for ζ are a) ζ is aligned with the radius vector to the point mass when ζ is the axis of least inertia, and b) ζ is orthogonal to the radius vector to the point mass and the normal to the orbit plane when ζ is the axis of greatest inertia.

Motions of ζ which remain close to each of the equilibrium positions are called *librations*. A closed form solution for the librational motion is not available. However, the system does have an energy integral, which will be used to establish the regions of stable motion of ζ about the equilibrium positions. This approach follows G. W. Hill,[1] who used the energy integral in the restricted problem of three bodies to establish the region of motion for the moon.

Energy Integral

Let xyz be an orthogonal triad with origin at the mass center 0 of the body, z normal to the orbit plane, and x in the initial direction of the point mass P. Let ξ point continuously from 0 to P. The angle ν between ξ and x is equal to nt where n is the orbit rate and t is the time. The orientation of the symmetry axis ζ is defined by the angles ϕ and λ; ϕ is measured from ξ to the projection of ζ on the orbit plane and λ is measured from the orbit plane to ζ. The angle ψ specifies the rotation of the body about ζ and completes the Eulerian set ϕ,λ, and ψ.

The kinetic energy of the body about 0 is

$$T = \tfrac{1}{2}A[\dot\lambda^2 + (\dot\phi + n)^2\cos^2\lambda] + \tfrac{1}{2}C[\dot\psi + (\dot\phi + n)\sin\lambda]^2$$

The gravity torque is derivable from the potential function

$$V = \tfrac{3}{2}n^2(C - A)\cos^2\lambda\cos^2\phi$$

Since neither T nor V depend explicitly on ψ, the system has the linear velocity integral

$$\dot\psi + (\dot\phi + n)\sin\lambda = s$$

where s, the spin rate, is the ζ component of the total angular velocity. The spin rate is a constant, which is taken as zero in this analysis.

The system also has an energy integral. Using the equation for $s = 0$ to eliminate $\dot\psi$, the energy integral becomes $R_2 - R_0 = \text{const}$ where

$$R_2 = \tfrac{1}{2}A(\dot\lambda^2 + \dot\phi^2\cos^2\lambda)$$

$$R_0 = -\tfrac{1}{2}An^2\sin^2\lambda - \tfrac{3}{2}(A - C)n^2(\sin^2\lambda + \cos^2\lambda\sin^2\phi)$$

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